Complex Analysis Playlist	**Reference Book**	**A Pathway to Complex Analysis by S Kumaresan**	
What is Complex Analysis about? -1		This is the first of a series of lectures. The aim is to give a bird's eye-view of a first course in complex analysis. This is the first of a series of lectures on a bird's eye view of a first course in Complex Analysis. The aim of the lectures is to demystify some of the `surprising' and `held-in-awe' results. We shall show how modern mathematics explains them. In fact, many themes such as topology developed because of the pioneering works of Cauchy, Riemann etc. This series is not for the faint-hearted* or for those whose aim is to get good grades in their semester exams. * Collins Dictionary: If you say that something is not for the faint-hearted, you mean that it is an extreme or very unusual example of its kind, and is not suitable for people who like only safe and familiar things. The series will be followed by a course on complex analysis soon	https://youtu.be/o8v_OK0mYqU
What is Complex Analysis about? - 2		In this session, we show how Cauchy theory attempts to prove the existence of local primitives of an holomorphic function. We then arrive at the Cauchy's theorem and its extension, and finally at the Cauchy integral formula	https://youtu.be/J7QZm16w5wg
What is Complex Analysis about? - 3		We state a result which says that functions defined by a Cauchy-type integral formula is analytic outside the path. Hence we conclude that any holomorphic function is analytic since it admits a Cauchy-type integral formula. In the second half of the lecture, we talk of the exponential function, its period, argument of a nonzero complex number. We state that there exists no continuous functions for the argument of elements of C^*. We indicate how this is the root cause for the so-called multi-valued functions. We conclude the lecture with the notion of logarithms.	https://youtu.be/SjYMCkFcn0Q
Continuous Argument – 1		The series of three lectures gives a rigorous understanding of the so-called multivalued maps. It points out the root cause for the multivaluedness is the argument of a complex number. It also shows how modern ideas, starting from Riemann deals with this in a satisfactory and practical way. In the first session, we recall the definition and properties of the exponential function on C. We define an argument of complex number and explain the significance of a continuous argument. We prove the non-existence of a continuous argument on C*. We try to salvage the situation by understanding the argument of a complex number of unit modulus.	https://youtu.be/s2Ng1e-E3cI
Continuous Argument - 2		We prove the existence of a continuous argument on a slit plane.	https://youtu.be/DLazsv5KiZQ
Continuous Argument - 3		We prove the existence of continuous (and hence holomorphic) logarithms on slit planes. We also indicate how to define power functions such as z raised to the power alpha (z^α).	https://youtu.be/WUk4CwZGluk

Power Series -1 : Radius of Convergence	Power series, radius and interval/disk of convegence	We motivate the power series by looking for a function f whose derivative is itself. Also, the Taylor series of an infinitely differentiable function. We prove a result on the radius of convergence of a power series. The proof is different from the standard ones found in textbooks which use the Hadamard's formula for the radius of convergence.	https://youtu.be/eNGi-V7s2CM
Power series 2: Term-wise differentiation of a power series		We recall the result which asserted the existence of the radius of convergence. We prove that the power series is uniformly convergent in B(a,r) where r is positive and less than the radius of convergence. We prove that the function defined by a (real) power series in its interval of convergence is differentiable and that the derivative is the function defined by the power series got by term-wise differentiation of the given power series.	https://youtu.be/bYfRWN8mhg4
Power series 3 - Complex Power Series		We prove that a complex power series defines a differentiable function on its disk of convergence. The derivative is again a power series got by term--wise differentiation of the first series. Note that the proof is valid in real case too and avoids the use of results such as the mean value theorem of real analysis.	https://youtu.be/Di02EalzCDQ
Power series 4: Hadamard's Formula for the Radius of Convergence		Since it is a standard result in any course, we derive the Hadamard's formula for the radius of convergence. For the benefit of the viewers, we have quickly recall the characterizing properties of the limsup of a bounded sequence of real numbers. It may be your worthwhile to review the concept of LimSup from https://youtu.be/I2_83xY15wc esp. From 00:10:05.	https://youtu.be/Hy1ZcWVvk6U
A Power Series Function is Analytic		Let f(z) be the sum of a convergent power series with B(0,R) as it disk of convergence. Then we give a direct simple proof of the result: f is analytic in B(0,R). We prove this by showing that the Taylor series of f at any z in B(0,R) converges in some disk B(z, r). This proof avoids any reference to double series or change in order of summation as found in the popular standard proof.	https://youtu.be/PzTvltPLXJk
Complex Analysis 1: Functions from R to C -1		As an important preliminary, we discuss the continuity, differentiability of function from an interval in R to C. Later we define the integral of such functions which are continuous and establish the analogues of fundamental theorems of calculus. and establish the basic integral inequality. We also give some nontrivial examples as applications of these results.	https://youtu.be/lYEf2LphTrE
Complex Analysis 2: Functions from R to C – 2		We establish the basic integral inequality and given an application to a uniformly convergent sequence of continuous functions. We then give two examples which illustrate various concepts and results of these lectures. The first one is to evaluate a real integral using the notion of the integral of a complex valued function. The second one is the Parseval identity which allows us to derive Liouville's theorem for power series convergent on all of C.	https://youtu.be/Y90lqQVWwXc

Complex Analysis 3: Holomorphic Functions – 1		We define thee differentiability of a function from C to C. We introduce the notion of holomorphic and entire functions. We state and prove a simple characterization of differentiability and use it to show that the set of functions differentiable at a point form a complex vector space. A few simple examples of differentiable functions are given.	https://youtu.be/pkSmm_Vrd1c
Complex Analysis 4: Holomorphic Functions – 2		We prove the chain rule and give applications.	https://youtu.be/lV6wk_s47w4
Complex Analysis 5: Holomorphic Functions – 3		We point out the mistake in the second example of the last session and show how to modify the argument to get a correct proof. We also indicate a second solution to the problem. We end of the session with a function which is defined by a Cauchy-type integral and show that it is holomorphic outside the domain of the integral. This example is very instructive and inspiring in a simple setting. It is never given in a typical course. But you will see that this gets you into the `real' complex analysis!	https://youtu.be/0LBir02PGgM
Complex Analysis 6: Holomorphic Functions -4: Cauchy Riemann Equations		We derive the Cauchy-Riemann equations. Wee also introduce the Cauchy-Riemann (differential) operator.	https://youtu.be/Ar6iAsEUQb0
Complex Analysis 7: Holomorphic Functions - 5 Cauchy-Riemann Equations -2		We prove that if the partial derivatives of u and v are also continuous, then the function f=u+iv is holomorphic.	https://youtu.be/UN0KVRYkf9g
Complex Analysis 8: Holomorphic Functions -6		We prove that the continuous inverse of an holomorphic function is holomorphic. We then apply it to branches of logarithms to conclude that the continuous logarithms are holomorphic. We derive the power series formula for a branch of the logarithm on a slit plane.	https://youtu.be/b2y3wsLivtc
Complex Analysis 9: Holomorphic Functions -7: What lies ahead?		We introduce the concept of an analytic functions and give an outline of a proof of the fact that a power series function is analytic. For complete details, the viewers may watch my videos on Double Series, especially, https://youtu.be/T3jV6OVTtTg A simple direct proof can be found in my video on "A Power Series Function is Analytic." https://youtu.be/PzTvltPLXJk	https://youtu.be/Jq3nu0cBgA0
A Power Series Function is Analytic		Let f(z) be the sum of a convergent power series with B(0,R) as it disk of convergence. Then we give a direct simple proof of the result: f is analytic in B(0,R). We prove this by showing that the Taylor series of f at any z in B(0,R) converges in some disk B(z, r). This proof avoids any reference to double series or change in order of summation as found in the popular standard proof.	https://youtu.be/PzTvltPLXJk
Complex Analysis 10: Path Integrals –1		We define the notion of piecewise C^1 paths, give some typical and most often needed examples. We also talk of juxtaposition/concatenation of paths.	https://youtu.be/8Vomcz37Vhc
Complex Analysis 11: Path Integrals -2		We define the integral of a continuous function along a path. We give a variety of examples, as the concept of path integrals is an important tool for almost all results of complex analysis. We draw attention to some of the beginners' mistakes.	https://youtu.be/pxE1rbWYapI

Complex Analysis 12: Path Integrals -3		We discuss an important class of path integrals. We state and prove various properties and results on path integrals. We end the session with (what I call) the fundamental theorem of path integrals.	https://youtu.be/FurTJoIQQAA
Complex Analysis 13: Path Integrals -4		We review the fundamental theorem of path integrals and revisit some of the earlier examples. We also redo the most important path integral (of winding number of a circle, a term not used in the video!) using logarithms. Hopefully, it will give a glimpse of how to use such multi-valued maps!	https://youtu.be/cvmk3GQ7r2w
Complex Analysis 14: Path Integrals–5		We derive the standard and useful estimate (known as the ML inequality) for a path integral. We give theoretical applications. We prove a very important result that a continuous function on a connected open set has a primitive iff its path integral on any closed path in the open set is zero. Go through this video at least twice, as almost all results in this session will be repeatedly used.	https://youtu.be/o_K3ufReR2Q
Complex Analysis 15: Path Integrals -6: Cauchy Type Integrals		In the last session of the series on path integrals, we generalize the Cauchy type integrals introduced earlier. We prove functions defined by Cauchy type integral are analytic in the complement of the track of the path. We also show that a circle (with the standard parametrization) winds around any point in the area enclosed by the circle. Note that this generalizes the earlier result which established this when the point was the centre of the circle.	https://youtu.be/kFLCbqgo7DU
Complex Analysis -16: Cauchy Goursat Theorem for Triangles		We prove Cauchy Goursat Theorem for Triangles	https://youtu.be/80w6PYLJE4E
Complex Analysis -17: Cauchy's Theorem for Star-shaped Doma		Let U be an open start-shaped set. Let F be holomorphic on U. The the path integral of f over any closed path in U is zero. This is the most fundamental result of Cauchy theory. As a consequence, we show that any holomorphic function has local primitives,	https://youtu.be/dqLBtQChn1k
Complex Analysis -18: Cauchy Theory-3		We prove an extension of Cauchy's theorem, derive the Cauchy Integral Formula (CIF) and prove the analyticity of a holomorphic function.	https://youtu.be/iDW3muUxWYc
Complex Analysis -19 Cauchy Theory-4		We prove the Cauchy Integral Formula for derivatives (CIFD). As an application, we prove Liouville's theorem. We point out a common misconception many beginners in the subject have about Lioville's theorem.	https://youtu.be/cbPL5UWJSnc
Complex Analysis 20: Cauchy Theory 5		We review Cauchy Integral Formula for Derivatives (CIF-D) and derive the Cauchy estimates. We give some typical applications.	https://youtu.be/wVIigpJW6UU
Cauchy Theory: Summary		We summarize the major steps which led us to CIF. Since the development of the final result is what I call boot-strap argument, it is good for beginners to review the key steps without getting into the details.	https://youtu.be/wdei_9Eyvm8
Complex Analysis: 21 (Main Results on) Holomorphic Functions – 1		We deal with Moera's theorem and give some typical applications, all of which are very important. We also state and prove Riemann's theorem on removable singularity to bring out the power of Morera's theorem.	https://youtu.be/ORkDahEUqew

Complex Analysis: 22 (Main Results on) Holomorphic Functions – 2		We prove the identity theorem and its applications.	https://youtu.be/8MV79Fc5BFU
Complex Analysis: 23 (Main Results on) Holomorphic Functions - 3 (Maximum Principle)		We prove the maximum modulus principle.	https://youtu.be/dpjOC6chYQo
Complex Analysis: 24 (Main Results on) Holomorphic Functions - 4 (Minimum Principle)		We prove thee minimum modulus principle and derive the open mapping theorem.	https://youtu.be/vUmKYVi1aXw
Complex Analysis: 25 (Main Results on) Holomorphic Functions – 5		We prove Weierstrass theorem on local uniform convergence of holomorphic functions. We show that an open subset of C is connected iff the ring of holomorphic functions on U is an integral domain. We also prove the existence of holomorphic logarithms of non-vanishing functions on star-shaped open sets.	https://youtu.be/lInyYze5Fes
Complex Analysis - 26 Extended Complex Plane -1		We introduce the notion of point at infinity to arrive at the extended complex plane. Since the viewers are likely to have learnt basic topology, we talk of one point compactification and show that the extended complex plane is a compact Hausdorff space.	https://youtu.be/Q0A72lx3nSI
Complex Analysis - 27: Extended Complex Plane-2		We discuss when a function defined in an open set containing infinity is holomorphic on the domain. This session is a stepping stone to manifolds and Riemann surfaces in the modern setting.	https://youtu.be/2XoIhHPrAmE
Complex Analysis-28: Extended Complex Plane – 3		We define holomorphic functions on open sets in the extended complex plane. We end the lecture with a proof of the fact that any holomorphic (complex valued) function on the extended plane is a constant.	https://youtu.be/Wqxk8DenLwk
Complex Analysis - 29: Isolated Singularities -1		We define isolated singularities and classify them by the behaviour of the function as it approached the isolated singularity. We give a (second) proof of the Riemann's theorem of removable singularity. We define the order of an isolated zero of a holomorphic function and show how it gives rise to a pole of the reciprocal function.	https://youtu.be/egkR-ygIlyY
Complex Analysis - 30: Isolated Singularities – 2		We have a closer look at poles. We give various characterizations in terms of the representation or the behaviour of the function at a pole of order m. We also derive the Laurent series expansion of the function around a pole. This is not found in many books.	https://youtu.be/5Nbk2MEKHt4
Complex Analysis - 31: Isolated Singularities – 3		We look at essential singularities. We prove Casaroti Weierstarss theorem on essential singularities. We characterize polynomials among the class of entire functions using the notion of essential singularity.	https://youtu.be/8mPdUehm8UM
Complex Analysis- 32: Laurent Series - 1		We prepare the ground for the Laurent series by staring a look at an infinite series indexed by negative integers, then a power series with negative powers, its domain of convergence differentiability etc. Then we look at a power series index by the set of integers, their convergence, differentiability, existence /non-existence of a primitive. This investigation brings out the important of the coefficient of 1/z-a.	https://youtu.be/chYYKTGc9JQ

Complex Analysis- 33: Laurent Series - 2		We state and prove the Leibnitz rule of differentiation under the integral sign for complex valued functions. We prove the Cauchy integral formula for an annular region and arrive at what I call the Laurent Decomposition.	https://youtu.be/uDtZJ6gNhTg
Complex Analysis - 34: Laurent Series - 3		We derive Cauchy Integral Formula for an annulus and prove the existence and uniqueness of the Laurent decomposition. We then arrive at the Laurent series, prove its uniqueness and the Cauchy estimates.	https://youtu.be/Qng16z2pAlI
Complex Analysis - 35: Laurent Series - 4		We work out an instructive example of computing the Laurent decomposition and the Laurent series of a function. In stead of working too many examples, I chose one telling example that illustrates the concepts and techniques.	https://youtu.be/eWJCPjsNbu8
Complex Analysis - 36: Laurent Series - 5		We characetrize the isolated singularities in terms of the Laurent series. Most often one derives the Laurent series and then classify isolated singularities using these characterizations	https://youtu.be/TVMPzxFPGks
Complex Analysis 37: Winding Numbers		We define winding numbers and establish their important properties	https://youtu.be/4H8dFlo0ZLY
Complex Analysis - 38: Global Version of Cauchy' s Theorem -1		This is the first of the series of 3 lectures on Dixon's proof of the global/Homology version of Cauchy's theorem.	https://youtu.be/GXa4J85uXvw
Complex Analysis -39: Global Version of Cauchy's Theorem -2			https://youtu.be/Vnck7zZnK5U