A Diagnostic Test

S. Kumaresan
School of Math. and Stat.
University of Hyderabad
Hyderabad 500046
kumaresa@gmail.com

Note: In True or False type questions, if you are not absolutely sure of the answer, then write "IDNK" (for "I do not know"). This will help us identify your problem areas.

Logic

1. Consider the sentence: "Every human being is clever." If this is false, what does it mean?
2. There exists a tree in this campus, which is fruit-bearing. If this is false, what does it mean?
3. I want to buy a pen which is cheap and good. The shop keeper shows me 10 pens. I do not find any pen which meets my requirements. What does this mean?
4. Let $\emptyset \neq A \subset \mathbb{R}$. If I say 17 is not an upper bound of A, what does it mean?

Set Theory

1. If $A, B \subset X$, and if $A \subset B$, what can you say about their complements A', B' (or $X \setminus A$ and $X \setminus B$)?
2. Let $f\colon X \to Y$ be a function, and $A, B \subset X$. What is the relation between $f(A \cup B)$, $f(A)$ and $f(B)$?
3. Given $f\colon X \to Y$ and $B \subset Y$, can you define $f^{-1}(B)$?
4. Given a finite set A with 100 elements, consider the sets $X = P(A)$, the power set of A and $Y = A \times A$. Which of X, Y has more elements?
5. True or False: The composition of two one-one (injective) functions is one-one?
6. Given two finite sets A and B with 10 and 100 elements, consider the set X of all functions from A to B. How many elements are there in X?
7. Give a bijection from the set of natural numbers $\mathbb{N}$ to the set of positive odd integers.

8. True or False: Equality relation on a set is an equivalence relation.

Real Analysis

1. What is the largest element (if it exists) of the open interval $(0, 1)$?
2. If (x_n) is an increasing sequence of real numbers, what does it mean to say that it is a bounded sequence?
3. If $a \in A \subset \mathbb{R}$ is an upper bound of A. What is the relation between a and the supremum or the least upper bound of A?
4. Draw the graph of the function $f(x) = |x - 1|$ for $x \in \mathbb{R}$.
5. Let $f\colon \mathbb{R} \to \mathbb{R}$ be given by $f(x) = |x + 1|$. Is it differentiable on $\mathbb{R}$?
6. True or false: If $x, y \in \mathbb{R}$ with $x < y$ then $\frac{1}{y} < \frac{1}{x}$.
7. True or false: There exists a differentiable function $f\colon [0, 1] \to \mathbb{R}$ such that $f(0) = 4$, $f(1) = 1$ and $f'(x) > 0$ for all $x \in [0, 1]$.
8. True or false: The function f defined on the set $\mathbb{R}^*$ of nonzero real numbers by $f(x) = 1$ if $x > 0$ and $f(x) = -1$ if $x < 0$ is continuous.
9. True or false: The function f defined on the set $\mathbb{R}^*$ of nonzero real numbers by $f(x) = 1$ if $x > 0$ and $f(x) = -1$ if $x < 0$ is differentiable.
10. True or false: every real valued continuous function on [2,20] is bounded.
11. True or false: There exists a continuous function $f\colon [0, 2] \to \mathbb{R}$ which is **onto**.

Linear Algebra

1. True or false: If S is a subset of a vector space such that the null (or zero) vector lies in S, then S is linearly dependent.
2. True or false: the set $\{(1, 2, 3), (-1, 2, 3), (3, 4, 5), (-1 - 1, -1)\}$ is linearly dependent in $\mathbb{R}^3$.
3. True or false: The map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (5x + 3y, x - y + 1)$ is linear.
4. True or false: If W is a vector subspace of a finite dimensional vector space V, then $\dim V \geq \dim W$.

General

1. Which is bigger: 10000^2 or 2^{10000}?