A Dignostic Test in Logical Thinking/Writing

The following sentences are jumbled version of the statement of a theorem and its proof. Identify the theorem and put the sentences in order so that we get the statement of the theorem and its proof as in a text-book.

1. Let $\varepsilon > 0$ be given.
2. Assume that $\sum_n C_n$ is convergent.
3. Let $f_n \colon X \to \mathbb{R}$ be a sequence of functions such that there exists $C_n > 0$ with $|f_n(x)| \leq C_n$ for all $x \in X$ and $n \in \mathbb{N}$.
4. Hence (s_n) is Uniformly Cauchy on X.
5. Since $\sum_n C_n$ is convergent, by the Cauchy criterion in the theory of infinite series, for the given $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $n > m \geq N$ implies that $\sum_m^n C_k < \varepsilon$.
6. It is enough to show that the sequence $(s_n(x))$ of partial sums of the series is uniformly Cauchy.
7. Then the series $\sum_n f_n(x)$ is uniformly convergent on X.
8. We have, for $n > m \geq N$, $|s_n(x) - s_m(x)| \leq \sum_{k=m}^n |f_k(x)| \leq \sum_{k=m}^n C_k < \varepsilon$ for all $x \in X$.

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The following sentences are jumbled version of the statement of a theorem and its proof. Identify the theorem and put the sentences in order so that we get the statement of the theorem and its proof as in a text-book.

1. Since f is increasing, $f(x_1) < f(x_2)$.
2. We claim that any strictly increasing function $f\colon \mathbb{R} \to \mathbb{R}$ is one-one.
3. By law of trichotomy, either $x_1 < x_2$ or $x_2 < x_1$.
4. If $x_2 < x_1$, we argue similarly to obtain $f(x_2) < f(x_1)$.
5. Let $x_1 \neq x_2$ be two real numbers.
6. A function $f\colon (a, b) \subset \mathbb{R} \to \mathbb{R}$ is said to be strictly increasing if for any pair $x_1, x_2 \in \mathbb{R}$ with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.
7. Hence the claim is proved.
8. Assume first that $x_1 < x_2$.
9. Hence $f(x_1) \neq f(x_2)$.
10. We have therefore shown that for any two distinct real numbers $x_1 \neq x_2$, we have $f(x_1) \neq f(x_2)$.

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